

# trig substitution cheat sheet

**Trig substitution cheat sheet** is an essential tool for students and professionals tackling integrals involving square roots of quadratic expressions. Trigonometric substitution is a technique that simplifies complex integrals by substituting trigonometric identities, making the integration process more manageable. This article will delve into the fundamentals of trigonometric substitution, provide a comprehensive cheat sheet, and illustrate its applications through examples.

## Understanding Trigonometric Substitution

Trigonometric substitution is primarily used when dealing with integrals that include expressions of the form:

- $\sqrt{a^2 - x^2}$
- $\sqrt{a^2 + x^2}$
- $\sqrt{x^2 - a^2}$

The primary goal of this technique is to transform these square root expressions into simpler trigonometric forms, allowing for straightforward integration.

## When to Use Trigonometric Substitution

You should consider using trigonometric substitution in the following scenarios:

1. Integrals involving square roots: When the integrand contains a square root of a quadratic expression, trigonometric substitution can often simplify the integral.
2. Complex fractions: If the integral contains a fraction with a square root in the denominator, trigonometric identities can simplify the expression.
3. Limits of integration: If the definite integral has limits that correspond conveniently to angles, trigonometric substitution can be particularly useful.

## Common Trigonometric Substitutions

Below is a cheat sheet outlining the standard trigonometric substitutions for specific types of square roots:

## 1. Substitutions for $\sqrt{a^2 - x^2}$

For integrals involving  $\sqrt{a^2 - x^2}$ , use the substitution:

- $x = a \sin(\theta)$
- Then,  $dx = a \cos(\theta) d\theta$
- This leads to  $\sqrt{a^2 - x^2} = a \cos(\theta)$

Example:

$$\int \sqrt{a^2 - x^2} \, dx$$

Substituting  $x = a \sin(\theta)$ :

$$= \int a \cos(\theta) \cdot a \cos(\theta) d\theta = a^2 \int \cos^2(\theta) d\theta$$

## 2. Substitutions for $\sqrt{a^2 + x^2}$

For integrals involving  $\sqrt{a^2 + x^2}$ , use the substitution:

- $x = a \tan(\theta)$
- Then,  $dx = a \sec^2(\theta) d\theta$
- This leads to  $\sqrt{a^2 + x^2} = a \sec(\theta)$

Example:

$$\int \sqrt{a^2 + x^2} \, dx$$

Substituting  $x = a \tan(\theta)$ :

$$= \int a \sec(\theta) \cdot a \sec^2(\theta) d\theta = a^2 \int \sec^3(\theta) d\theta$$

## 3. Substitutions for $\sqrt{x^2 - a^2}$

For integrals involving  $\sqrt{x^2 - a^2}$ , use the substitution:

- $x = a \sec(\theta)$

- Then,  $dx = a \sec(\theta) \tan(\theta) d\theta$
- This leads to  $\sqrt{x^2 - a^2} = a \tan(\theta)$

Example:

$$\int \sqrt{x^2 - a^2} \, dx$$

Substituting  $x = a \sec(\theta)$ :

$$= \int a \tan(\theta) \cdot a \sec(\theta) \tan(\theta) d\theta = a^2 \int \tan^2(\theta) \sec(\theta) d\theta$$

## Steps for Trigonometric Substitution

To effectively use trigonometric substitution, follow these systematic steps:

1. **Identify the integral:** Look for square root expressions of the forms discussed above.
2. **Choose the right substitution:** Based on the form of the expression, select the appropriate trigonometric substitution.
3. **Differentiate:** Compute  $dx$  in terms of  $d\theta$ .
4. **Rewrite the integral:** Substitute  $x$  and  $dx$  into the integral, transforming it into a trigonometric integral.
5. **Integrate:** Solve the resulting integral using trigonometric identities.
6. **Back-substitute:** Replace  $\theta$  with  $x$  using the original substitution to express the solution in terms of  $x$ .

## Examples of Trigonometric Substitution

To solidify the understanding of this technique, let's work through a couple of examples.

## Example 1: Integrating $\sqrt{1 - x^2}$

Consider the integral:

$$\int \sqrt{1 - x^2} \, dx$$

1. Identify the form: This fits the  $\sqrt{a^2 - x^2}$  form where  $(a = 1)$ .
2. Choose substitution: Let  $(x = \sin(\theta))$ , hence  $(dx = \cos(\theta) \, d\theta)$ .
3. Rewrite the integral:

$$\int \sqrt{1 - \sin^2(\theta)} \cos(\theta) \, d\theta = \int \cos^2(\theta) \, d\theta$$

4. Integrate:

$$\int \cos^2(\theta) \, d\theta = \frac{1}{2}(\theta + \sin(\theta)\cos(\theta)) + C$$

5. Back-substitute:

Since  $(x = \sin(\theta))$ , then  $(\theta = \arcsin(x))$ :

$$\int \sqrt{1 - x^2} \, dx = \frac{1}{2} \left( \arcsin(x) + x\sqrt{1 - x^2} \right) + C$$

## Example 2: Integrating $\sqrt{x^2 + 4}$

Now consider the integral:

$$\int \sqrt{x^2 + 4} \, dx$$

1. Identify the form: This fits the  $\sqrt{a^2 + x^2}$  form where  $(a = 2)$ .
2. Choose substitution: Let  $(x = 2\tan(\theta))$ , hence  $(dx = 2\sec^2(\theta) \, d\theta)$ .
3. Rewrite the integral:

$$\begin{aligned} \int \sqrt{4 + 4\tan^2(\theta)} \cdot 2\sec^2(\theta) \, d\theta &= \int 2\sqrt{4\sec^2(\theta)} \cdot 2\sec^2(\theta) \, d\theta \\ &= 4 \int 2\sec^3(\theta) \, d\theta \end{aligned}$$

4. Integrate: Use the formula for  $(\int \sec^3(\theta) \, d\theta)$ :

$$\int$$

$$\int \sec^3(\theta) d\theta = \frac{1}{2}(\sec(\theta)\tan(\theta) + \ln |\sec(\theta) + \tan(\theta)|) + C$$

5. Back-substitute:

Replace  $\theta$  back in terms of  $x$ :

$$= 4\left(\frac{1}{2} \left( \sec(\theta)\tan(\theta) + \ln |\sec(\theta) + \tan(\theta)| \right) + C\right)$$

Finally, replace  $\sec(\theta)$  and  $\tan(\theta)$  with expressions in  $x$ .

## Conclusion

In conclusion, the **trig substitution cheat sheet** is a vital resource for simplifying integrals that contain square roots of quadratic expressions. By mastering the substitutions and following the systematic steps outlined in this article, students and professionals can tackle complex integrals more efficiently. Practicing these techniques will enhance your problem-solving skills and deepen your understanding of calculus.

## Frequently Asked Questions

### What is trig substitution and when is it used?

Trig substitution is a technique used in calculus to simplify integrals involving square roots of quadratic expressions. It is particularly useful when dealing with integrals that cannot be easily solved through basic methods.

### What are the common trig identities used in trig substitution?

The common trig identities used in trig substitution include  $\sin^2(\theta) + \cos^2(\theta) = 1$ ,  $\tan^2(\theta) + 1 = \sec^2(\theta)$ , and  $1 + \cot^2(\theta) = \csc^2(\theta)$ . These identities help simplify the expressions during substitution.

### What are the common types of substitutions in trig substitution?

The common types of substitutions in trig substitution are: 1)  $x = a \sin(\theta)$  for  $\sqrt{a^2 - x^2}$ , 2)  $x = a \cos(\theta)$  for  $\sqrt{x^2 - a^2}$ , and 3)  $x = a \tan(\theta)$  for  $\sqrt{a^2 + x^2}$ .

### How do you choose the correct substitution for an integral?

To choose the correct substitution for an integral, identify the form of the expression under the square root. For example, if it resembles  $\sqrt{a^2 - x^2}$ , use  $x = a \sin(\theta)$ . If it resembles  $\sqrt{x^2 - a^2}$ , use  $x = a \sec(\theta)$ , and for  $\sqrt{a^2 + x^2}$ , use  $x = a \tan(\theta)$ .

## What steps should be followed after making a trig substitution?

After making a trig substitution, follow these steps: 1) Rewrite the integral in terms of  $\theta$ , 2) Simplify the integrand using trigonometric identities, 3) Change the differential  $dx$  to  $d\theta$  using the derivative of the substitution, and 4) Integrate and then convert back to  $x$  using the original substitution.

## Are there any common mistakes to avoid when using trig substitution?

Common mistakes include forgetting to change the limits of integration if dealing with definite integrals, failing to simplify the integrand properly, and neglecting to convert back to the original variable after integration.

## Trig Substitution Cheat Sheet

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